A photon's four-dimensional shape defines its ratio of hypervolume to (momentum-wavelength). The following shows a numerical verification of the hyperscaling factor with a comparison to the triangular case:

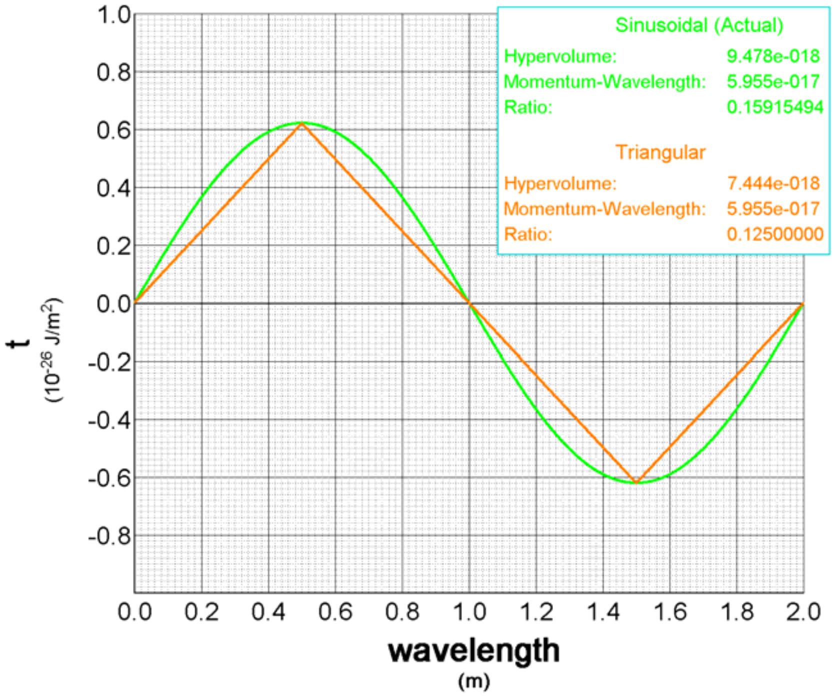

Figure (8.2) Numerical verification of the photon hyperscaling factor

Unit hypervolume can now be written directly in terms of Planck's constant:

$$\Diamond_4 = \Diamond_\gamma = \frac{p_\gamma \lambda}{2\pi} = \frac{E_\gamma c\lambda}{2\pi} = \frac{hc^2}{2\pi} = \hbar c^2 \tag{8.14}$$

in units of J-m^2/s, using the Planck relation $E_\gamma\lambda = hc$. When $\Diamond_4$ is expressed in units of energy, (J-m), instead of momentum, it is equal to the product of the speed of light and the most common representation of Planck's constant in quantum physics:

$$\Diamond_4 = \frac{hc}{2\pi} = \hbar c \tag{8.15}$$

This is no coincidence.

The photon hyperscaling calculation is not dependent on the relationship between a photon's width or height; indeed it is not even dependent on its actual physical wavelength. The only criterion is it varies sinusoidally along one dimension. If a photon were physically long, its energy density would decrease but its total energy would remain constant. The same is true of its hypervolume.

Unit polarvolume, the constant that defines matter's quantization, follows from Equations (8.2) and (8.14):

$$\Diamond_q = \frac{hc}{4\pi} \tag{8.16}$$

in units of J-m.

8.4 REALITY'S FOUR-DIMENSIONAL SIZE

Equation (8.14) yields the universe's four-dimensional volume as $9.478017(10)^{-18}$ J-m^2/s. This value has the accuracy of Planck's constant, which is far better than 1 ppm. It can be converted directly into quadric meters by dividing by universal energy density and the speed of light:

$$\Diamond_4 = \frac{\hbar c}{\rho_U} \tag{8.17}$$

Unfortunately, universal energy density is not known to great precision, but using an estimate of $4(10)^{-10}$ J/m^3 (as derived later in Part IV) results in a unit hypervolume of ~$7.9(10)^{-17}$ m^4 and an absolute meter of ~0.1 mm.

The SI units of distance, time, and energy all have absolute values defined directly by the universe's four-dimensional size, a size that can be expressed in a number of equivalent ways:

- Using absolute units of $s_a m_a^3$: 1 s_a-m_a^3.
- Using SI units of meter^4: ~$7.9(10)^{-17}$ m^4.
- Using SI units of second-meter^3: ~$2.6(10)^{-25}$ s-m^3.
- Using SI units of joule-meter^2/second: $9.478017(10)^{-18}$ J-m^2/s.
- Using SI units of joule-meter: $3.161526(10)^{-26}$ J-m.

As our development proceeds, the most commonly used form will be the last, in J-m, owing to its simplicity and the generally low accuracy of universal energy density measurements.

8.5 QUANTUM SUPERPOSITION

It might seem odd that individual quanta have sizes comparable to that of the universe. There are millions of photons and at least a few particles in every cubic meter of space. This