EINSTEIN'S NONPHYSICAL GEOMETRY

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Abstract: General relativity is a major driving force in the pursuit of modern cosmology. In this paper the author argues that its geometry should not be interpreted as the literal physical representation of space and time.

I. GENERAL RELATIVITY

The General Theory of Relativity is lauded not only for its predictive power but also for its mathematical eloquence. The foundation of this theory is the ultimate expression of simplicity. It originates from our inability to experimentally distinguish:

- The simultaneity of two events separated in space.
- The difference between an accelerating observer and the presence of a gravitational field.

The complete mathematical rendering of this concept is too involved to present as an overview, but a brief description of one of its metrics follows.

DIFFERENTIAL GEOMETRY

The Lorentz transform demonstrates a relationship between measured space and time in terms of a single dimension. The generalization of this concept to three dimensions is:

$$ds^2 = c^2dt^2 - \left(dx^2 + dy^2 + dz^2\right) \qquad (1)$$

where a differential interval of space-time *ds* has competing components of space and time.

Since space and time are the primary components and this paper is limited to radially symmetric fields, this can be simplified to:

$$ds^2 = c^2dt^2 - dr^2 \qquad (2)$$

Minkowski space is the mathematical formalization of the relativity of simultaneity. Two events in space-time are separated by a certain amount of distance, a certain amount of time, or a combination of both. They are considered simultaneous if the time difference between them is zero ($dt = 0$). In this case the space-time distance between them is described strictly in terms of space. This is called the *proper distance*, and it is equal to:

$$dL = \sqrt{-ds^2} \tag{3}$$

Substitution of Equation (2) into this expression with ($dt = 0$) yields a difference of space:

$$dL = dr \tag{4}$$

The maximum passage of time occurs when there is no change in space. This is called the *proper time*, and is equal to:

$$d\tau = \sqrt{\frac{ds^2}{c^2}} \tag{5}$$

when ($dr = 0$). Proper distance represents two events simultaneous in time; proper time represents two events simultaneous in space.

Although time is often interpreted in relativity theory as a fourth dimension external to space, the Minkowski metric actually demonstrates this is not true. If time were truly an extension of space then the distance between any two events would have the form:

$$dw^2 = c^2dt^2 + dr^2 \tag{6}$$

where differences in space and time compound and compliment each other. This is not the case. A difference of time occurs at the expense of a difference in distance, *because time is a contextual difference of space*.[1] This is the basis of Equation (2) above:

$$ds^2 = c^2dt^2 - dr^2$$

Distance and space-time distance for four events are shown in the figure below:

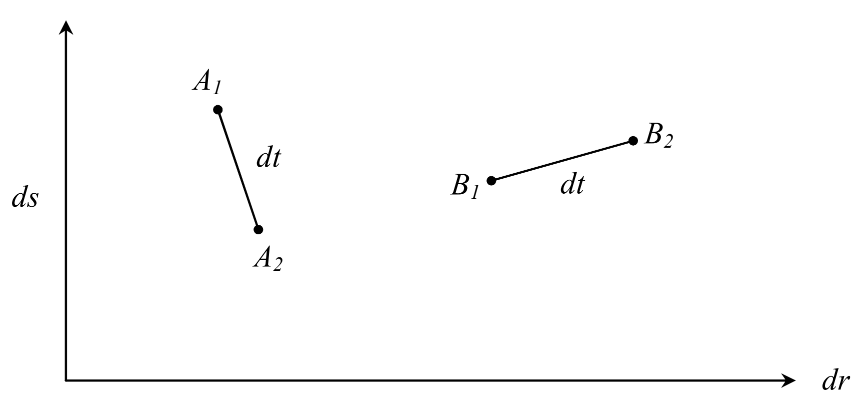

Figure (1) Minkowski space depicts time as a dependent dimension

If two events such as $\mathbf{A}_1$ and $\mathbf{A}_2$ are close in space, yet distant in space-time, then there is a large temporal difference between them. Conversely, if two events $\mathbf{B}_1$ and $\mathbf{B}_2$ are about the same distance apart in either space or time, then the space-time distance between them is quite small even though they may quite remote from each other. What this means is all events along a photon's path are virtually indistinguishable because the *ds* interval is zero. Minkowski space is a direct reflection of *dimensional closure*, which expresses the interdependence (closure) between the space and time of the universe.[2]

II. THE SCHWARZSCHILD METRIC

The General Theory of Relativity describes how the metric of Equation (2) is distorted in the vicinity of massive objects. Space is typically portrayed as being stretched radially toward the center of the field as it is compressed laterally, and time experiences a general dilation. Although there is no brief explanation of the derivation of the equations of General relativity, many of their solutions are fairly straightforward. The simplest is the field around a non-rotating spherical object. It is called the *Schwarzschild metric*:[3]

$$ds^2 = \left(cdt\sqrt{1-\frac{2GM}{rc^2}}\right)^2 - \left(\frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}}\right)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2 \tag{7}$$

where *ds* is a differential unit of length in four-dimensional space-time as presented above and θ and ϕ are polar coordinates in space.

This can be further simplified by restricting it to the space-time extending normal to the surface of the spherical body ($d\theta = d\phi = 0$). The metric along any radial distance from the center of the object is given by:

$$ds^2 = c^2dt^2\left(1-\frac{2GM}{rc^2}\right) - \frac{dr^2}{\left(1-\frac{2GM}{rc^2}\right)} \tag{8}$$

This will be referred to as the *radial Schwarzschild metric*.

Note that this is merely a mass-modified version of the undistorted metric of Equation (2):

$$ds^2 = c^2dt^2 - dr^2$$

What Equation (8) says is time contracts and distance expands in the presence of a gravitational field. Unlike an attenuating radial distortion caused by a centralized stress within a solid material, Equation (8) only relates differential lengths at various positions within a distorted metric. There is no absolute metric. It can, however, be converted into a field of spatial deflection for comparative purposes. The magnitude of spatial distortion in the Schwarzschild metric can be isolated by restricting it to a coordinate system simultaneous in time ($dt = 0$).

Applying this constraint to Equation (8) results in:

$$ds^2 = -\frac{dr^2}{\left(1 - \frac{2GM}{rc^2}\right)} \tag{9}$$

Here the only differences between its points are purely spatial. Applying Equation (3) yields a distance metric of the form:

$$ds = dL = \frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}} \tag{10}$$

The distended length between two radii is the integral of the dL length element:

$$L = \int_{r=R_1}^{R_2} dL = \int_{r=R_1}^{R_2} \left(\frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}} \right) \tag{11}$$

The change in radial length per radial length caused by a central mass M is given by:

$$\frac{L - \Delta R}{\Delta R} = \frac{\int_{r=R_1}^{R_2} \left(\frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}} \right) - (R_2 - R_1)}{(R_2 - R_1)} \tag{12}$$

In the limit $(R_2 - R_1) \rightarrow dr$ this becomes what will be referred to as *relativistic slope*:

$$\delta_r'(r) = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} - 1 \tag{13}$$

where the term δ_r will be used to denote a relativistic difference of space along space, and will be referred to as *relativistic deflection*. When $r \to R_S$, the Schwarzschild radius ($2GM/c^2$), relativistic slope goes to positive infinity. When r is large in comparison to R_S, Equation (13) resolves to:

$$\delta_r'(r) \to \frac{GM}{rc^2} \tag{14}$$

To obtain the magnitude of the deflection of space along space from Equation (14), integrate with respect to r:

$$\int \delta_r'(r)dr = \delta_r(r) + C = \left(\frac{GM}{c^2}\right)\left(\ln(r) - \ln(R_{\{0\}})\right) \qquad \{r >> R_S\} \tag{15}$$

where the constant of integration is defined in terms of the logarithm of a fixed radius $R_{\{0\}}$. $R_{\{0\}}$ will be called the *zero deflection radius*, for when r is equal to this radius, relativistic deflection is equal to zero. The deflection in this expression is positive as a matter of convention.

Equation (15) demonstrates a curious aspect of General Relativity. It describes a spatial deflection field *increasing without bound with distance*. Another way to look at this is in terms of the deflection at some radius R relative to infinity:

$$\delta_r(R >> R_S) = \int_{r=R}^{\infty} \delta_r' dr = \int_{r=R}^{\infty} \frac{GM}{rc^2} dr = \frac{GM}{c^2}\left(\ln(\infty) - \ln(R)\right) = \infty \tag{16}$$

Since the relativistic deflection of space does not converge, the actual magnitude of deflection at any location in the field is undefined with respect to infinite range. If mass is indeed the agent of space-time's distortion, the distortion that a finite mass produces ought to attenuate to zero at infinite distance. Equation (16) shows that this is not the case. The spatial deflections of general relativity do not converge with increasing values of r. *The presence of a finite mass in space adds an infinite amount of distance between any two infinitely remote points in its field*. Or equivalently, since spatial displacement also represents volumetric displacement, the presence of a finite mass deflects an infinite amount of space. This is a physically nonsensical situation, of which the only counterargument is that space-time is not a physical *thing*. Such semantics provide no more than a feeble defense, tantamount to saying that distance and time are not physical (real) things. If space can be curved, it can be deflected, and a spatial deflection is no more or less real than the space it deflects.

The following graph shows relativistic deflection from Equation (16) as a function of radius in the vicinity of the sun, where zero deflection is defined at its surface:

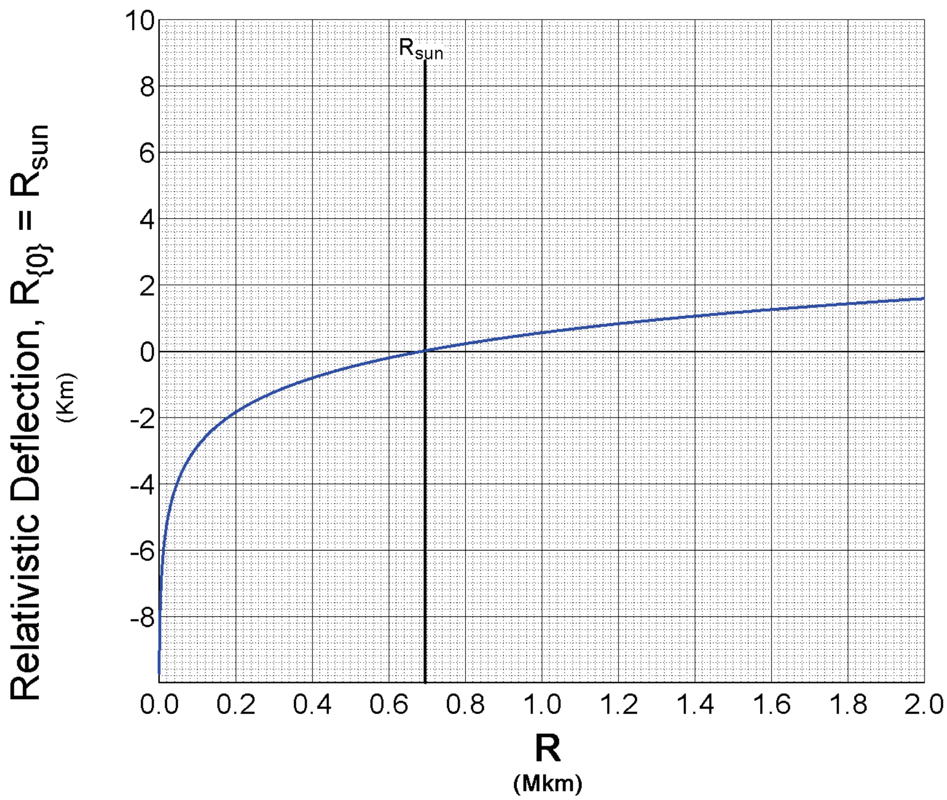

Figure (2) Relativistic deflection as a function of radius

Deflection is negative in the sun's interior and decreases until our condition of ($r >> R_S$,) is no longer valid. Beyond the sun's surface, deflection is positive, increasing with no limit to positive infinity. Choosing different values for $R_{\{0\}}$ moves the radius of zero deflection, *but the Schwarzschild metric is non-convergent regardless of the value of* $R_{\{0\}}$. Yet this curious property is not the only compelling reason why the distortion described by the General Theory has no correspondence to actual spatial displacement. *Relativistic deflection is not an accurate portrayal of physical spatial deflection because it contains no causative agent for its variation with distance*. In other words, distortional fields attenuate with distance because their effect is diluted by the object of their distortion. A mass distorts space-time, so the magnitude of the distortion it causes ought to attenuate with distance from a mass in direct proportion to the increasing volume of space-time associated with said distance; space-time is the *cause* of the attenuation. This is not the case in the Schwarzschild metric because it has no physical reality.

The General Theory does not contain a single questionable or unreasonable assertion, and it results in a spectacularly accurate portrayal of gravitational interaction. But the geometry of its metric is inconsistent with physical reality. Gravitational phenomena are an indirect consequence of an underlying physical geometry; they do not directly define this geometry.

REFERENCES

[1] *Our Undiscovered Universe*, Terence Witt [Aridian Publishing, 2007], p. 70

[2] *Our Undiscovered Universe*, Terence Witt [Aridian Publishing, 2007], p. 56

[3] *Modern Astrophysics*, Bradley Carroll & Dale Ostlie [Addison Wesley Longman, 1996], p. 657